

Phase synchronization and noise-induced resonance in systems of coupled oscillators

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We study synchronization and noise-induced resonance phenomena in systems of globally coupled oscillators, each possessing finite inertia. The behavior of the order parameter, which measures the collective synchronization of the system, is investigated as the noise level and the coupling strength are varied, and hysteretic behavior is manifested. The power spectrum of the phase velocity is also examined and the quality factor as well as the response function is obtained to reveal noise-induced resonance behavior.

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I. INTRODUCTION

In recent years the networks of coupled nonlinear oscillators have attracted much attention: They serve as a prototype model for a variety of self-organizing systems in physics, chemistry, biology, and social sciences, and exhibit the remarkable phenomena of synchronization [1,2]. Among those the system with global coupling has been mostly studied both analytically and numerically, owing to analytical simplicity and some physical as well as biological applications [3–5]. In such a system of globally coupled oscillators, the effects of nonzero inertia and of noise as well as the effects of periodic driving on synchronization have been examined [6,7]. Of particular interest in the presence of noise is the possible amplification of the response of the system, arising from the interplay between the noise and driving [8]. Such stochastic resonance phenomena, having various potential applications, have received much attention [9–12]. Recently, the interesting possibility of stochastic resonance in systems without external periodic driving has been pointed out [13–15]. For example, noise-controlled resonance behavior in a periodic potential with constant driving has been discussed, and in the limit of low damping the inertia has been shown to play the role of a surrogate of external periodic driving [14]. While such noise-induced resonance behavior has been investigated in oscillator systems with relatively few degrees of freedom, typically single-oscillator systems, or in systems of excitable elements [16], the possibility of detecting the resonance behavior in a system of coupled (nonexcitable) oscillators has not been properly addressed.

The purpose of this paper is to examine whether such noise-induced resonance behavior can appear in a coupled oscillator system with many degrees of freedom. We thus consider the system of globally coupled stochastic oscillators, each possessing finite inertia, subject to constant driving force, and investigate the behavior of the order parameter, which measures the synchronization of the system, as the noise level and the coupling strength are varied. To understand the interplay of noise and driving force, giving rise to the possibility of noise-induced resonance behavior in the absence of periodic driving, we further consider the power spectrum of the phase velocity as the response to the driving force, and investigate both the zero-frequency (dc) component and the nonzero-frequency (ac) one. The dc component of the power spectrum, proportional to the squared time av-

erage of the phase velocity, measures the probability that the system, kicked by noise, eventually escapes out of a potential-well minimum. Namely, it describes the interwell transition. On the other hand, the ac component describes the intrawell oscillation behavior, which does not induce an escape out of the potential-well minimum. The order parameter is observed to decrease with noise, manifesting suppression of the synchronization, and to display hysteretic behavior with the noise level as well as with the coupling strength. Suppression of the synchronization is also reflected by the growth of the dc component of the power spectrum, corresponding to the dispersion of the mean phase velocity, with noise. On the other hand, it is found that the generalized susceptibility, related to the power spectrum via the fluctuation-dissipation theorem, increases first as the noise grows from zero, reaches its maximum at a finite noise level, and eventually decreases as the noise level is increased further. Such noise-induced effects are also observed in the quality factor at appropriate nonzero frequencies, suggesting the presence of intrawell resonance.

This paper consists of five sections: Section II introduces the system of coupled oscillators, each possessing finite inertia, subject to random noise and constant driving force. The self-consistency equation for the order parameter, which measures the collective synchronization in the system, is described. In Sec. III, the behavior of the order parameter with the coupling strength and the noise level is investigated, which manifests hysteretic behavior at low noise levels. Section IV is devoted to the investigation of the response of the phase velocity to the external driving force, focusing on the interplay between noise and driving. The power spectrum of the phase velocity is revealed to exhibit noise-induced resonance in appropriate regimes. Finally, a brief summary is given in Sec. V.

II. SYSTEM OF COUPLED OSCILLATORS

We begin with the set of equations of motion governing the dynamics of N coupled oscillators, the i th of which is described by its phase $\phi_i (i=1,2,\dots,N)$:

$$\mu \dot{\phi}_i + \phi_i + \frac{K}{N} \sum_{j=1}^N \sin(\phi_i - \phi_j) = \omega_i + \eta_i(t), \quad (1)$$

where μ represents the magnitude of the (rotational) inertia relative to the damping. The third term on the left-hand side of Eq. (1) denotes the global coupling with strength K/N , whereas the first and the second on the right-hand side represent the constant driving force and the random (thermal) noise, respectively. The driving force ω_i is distributed over the whole oscillators according to the distribution $g(\omega)$, which is assumed to be smooth and symmetric about $\omega=0$. The term $\eta_i(t)$ represents independent white noise with zero mean and correlations $\langle \eta_i(t) \eta_j(t') \rangle = 2T \delta_{ij} \delta(t-t')$, where the noise level $T(>0)$ plays the role of the ‘‘effective temperature’’ of the system. The set of equations of motion in Eq. (1) describes a superconducting wire network [17] and may also be regarded as the mean-field version of an array of resistively and capacitively shunted junctions, which serves as a common model for describing the dynamics of superconducting arrays [18]. In these cases, the constant driving force ω_i corresponds to the direct current bias.

Collective behavior of such an N -oscillator system is conveniently described by the complex order parameter

$$\Psi \equiv \frac{1}{N} \sum_{j=1}^N e^{i\phi_j} = \Delta e^{i\theta}, \quad (2)$$

where nonvanishing magnitude ($\Delta \neq 0$) indicates the emergence of collective synchronization and θ gives the average phase. Note that the synchronized state corresponds to the superconducting state with global phase coherence in the case of superconducting networks or arrays [17]. The order parameter defined in Eq. (2) allows us to reduce Eq. (1) to a *single* decoupled equation

$$\mu \dot{\phi}_i + \dot{\phi}_i + K\Delta \sin(\phi_i - \theta) = \omega_i + \eta_i(t), \quad (3)$$

where Δ and θ are to be determined by imposing self-consistency. Namely, the order parameter, defined in terms of the phase via Eq. (2), in turn determines the behavior of the phase via Eq. (3), which depends explicitly on Δ and θ . We then seek a stationary solution with θ being constant, which is possible due to the symmetry of the distribution of ω_i about zero. Redefining $\phi_i - \theta$ as ϕ_i and suppressing indices, we write the reduced equation of motion in the form

$$\mu \dot{\phi} + \dot{\phi} + K\Delta \sin \phi = \omega + \eta(t), \quad (4)$$

which depends explicitly on the magnitude Δ of the order parameter.

In the absence of noise ($T=0$), the self-consistency equation for the order parameter reads

$$\Delta = \left(\frac{\pi}{2} - \frac{\mu}{2} \right) g(0) K\Delta + \frac{4}{3} \mu g(0) (K\Delta)^2 + \frac{\pi}{16} g''(0) (K\Delta)^3 + O(K\Delta)^4. \quad (5)$$

If the distribution $g(\omega)$ is given, the collective behavior of the system can thus be obtained by solving Eq. (5). In general the quadratic term of order $(K\Delta)^2$ is known to induce hysteresis in the bifurcation diagram [6]. Accordingly, it has been concluded that the nonzero inertia tends to induce hysteresis in the bifurcation diagram of the system [7].

The self-consistency equation for the order parameter in the presence of noise, particularly at such high temperatures that $K\Delta/T \ll 1$, has also been considered, yielding [7]

$$\Delta = \Delta_+ \equiv \frac{\sqrt{cK(aK-1)}}{cK^2}, \quad (6)$$

with the coefficients given by the integrals

$$a = \int_{-\infty}^{\infty} d\omega g(\omega) \frac{T - \mu\omega^2}{2(T^2 + \omega^2)},$$

$$b = \int_{-\infty}^{\infty} d\omega g(\omega) \left[\frac{T + \mu(T^2 - \omega^2) - \mu^2\omega^2 T}{4(T^2 + \omega^2)^2} + \frac{\mu^3\omega^2 + 2\mu^2 T}{8(T^2 + \omega^2)} - \frac{6T + \mu(8T^2 - \omega^2) + \mu^2 T(8T^2 - \omega^2)}{8(T^2 + \omega^2)(4T^2 + \omega^2)} \right].$$

In this case collective behavior of the system has been obtained as follows: When $K < K_c \equiv 1/a$, only the null solution ($\Delta=0$) is possible. At $K=K_c$, on the other hand, the null solution loses its stability and the nontrivial solution Δ_+ , together with the unphysical solution $\Delta_- \equiv -\Delta_+$, emerges via a pitchfork bifurcation. Subsequently, it grows in a continuous manner $(a^2/\sqrt{c})(K-K_c)^{1/2}$ as K is increased beyond K_c [6,7].

III. PHASE SYNCHRONIZATION

In this section we present in detail the behavior of the order parameter with the coupling strength and the noise level. We have performed extensive numerical simulations on the equations of motion given by Eq. (1) at various noise levels and coupling strengths. The order parameter Δ has been computed from the definition given by Eq. (2), and its behavior depending on the coupling strength and the noise level has been examined. In simulations Eq. (1) has been integrated with discrete time steps of $\delta t = 0.001$, and for convenience, a semicircle distribution of radius $r=0.5$ has been chosen for $g(\omega)$. (We have also considered other types of distributions such as Gaussian, only to find no qualitative change.) In computing the order parameter, $N_t = 10^5$ time steps have been used while the data from the first 5×10^4 steps are discarded at each run. Both δt and N_t have been varied to confirm that the stationary state has been achieved. We have then computed the order parameter in the system of $N=2000$ oscillators, each having inertia $\mu=0.8$.

The obtained behaviors of the order parameter with the coupling strength and the noise level are displayed in Figs. 1 and 2. Figure 1(a) shows the behavior as the coupling strength K is varied with the noise level T kept fixed: Circles and squares describe the behavior of the order parameter as the coupling strength is increased and decreased, respectively. At zero noise ($T=0$), 20 independent runs have been performed with different initial configurations, over which averages are taken. The corresponding error bars have been

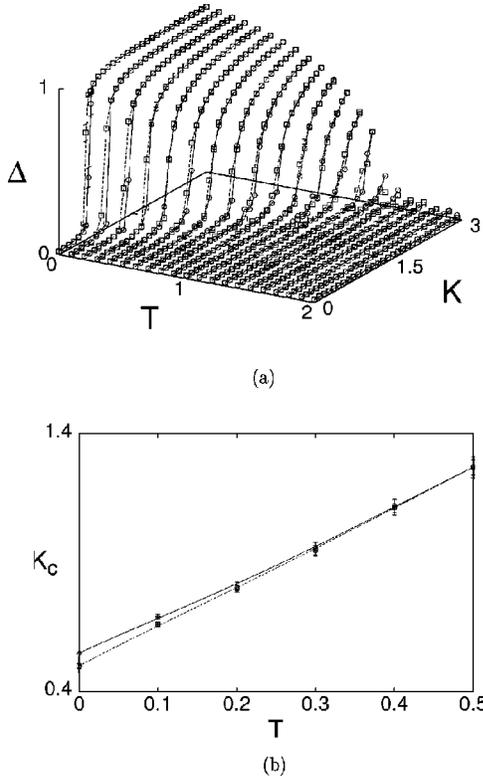


FIG. 1. (a) The order parameter as a function of the coupling strength K for various values of the noise level T . Circles and squares represent the data for increasing and decreasing the coupling strength, respectively, and the solid and dashed curves are merely guides to the eye. Hysteresis is manifested at zero noise and observed to weaken with the noise. (b) Critical coupling strength, beyond which synchronization sets in, is shown to increase with T . Notation is the same as that in (a), with the error bars estimated by the standard deviation. It is observed that noise in general suppresses both synchronization and hysteresis.

estimated by the standard deviations whereas those data points without explicit error bars have errors smaller than the size of the symbol. Note the hysteresis manifested at zero noise and weakening as the noise strength is increased from zero. These characteristic features as K is varied for a given T agree well with the results of Ref. [7]. Figure 1(b) shows that the critical coupling strength K_c , beyond which synchronization sets in, increases monotonically with the noise level T , demonstrating the suppression of synchronization by noise. Here K_c has been estimated by the value of the coupling strength at which the order parameter Δ first becomes nonzero to a precision of 10^{-1} . The circles and squares, again corresponding to the data for increasing and decreasing K , respectively, have been obtained from averages taken over ten independent runs with different initial configurations and the error bars estimated by the standard deviation. Thus, unlike the excitable system [16], noise-induced synchronization does not emerge here. Further, the hysteretic behavior, reflected by the difference in the critical coupling strength between the two cases, is revealed to diminish conspicuously as the noise level is increased.

In Fig. 2(a) the behavior of the order parameter with the noise level T for fixed coupling strength K is displayed. Here circles and squares represent the data for increasing and de-

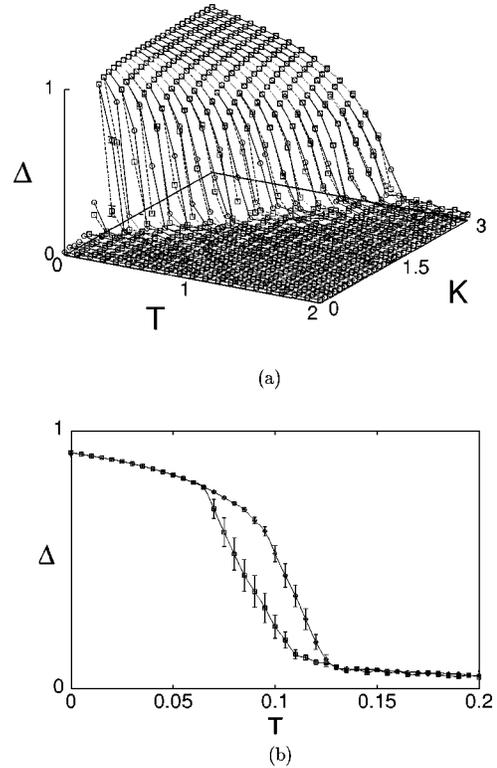


FIG. 2. (a) The order parameter as a function of the noise level T for various values of the coupling strength K . Circles and squares represent the data for increasing and decreasing the noise level, respectively, and the solid and dashed curves are merely guides to the eye. (b) Behavior of the order parameter with the noise level at coupling strength $K=0.7$, with the same notation as in (a). Manifested is the hysteretic behavior as the noise level is varied.

creasing T , respectively, and the typical error bars, estimated by the standard deviation obtained from 20 independent runs with different initial configurations, are shown on the data points at $K=0.7$. Figure 2(b) displays the detailed behavior with the noise strength for the coupling strength $K=0.7$, again manifesting the hysteresis. Note that the hysteresis is most conspicuous for the coupling strength around this value, decreasing as the coupling strength is increased. At weak coupling strengths, the system is not synchronized ($\Delta=0$), giving no hysteresis. It is thus concluded that the system exhibits quite generally hysteretic behavior as either the coupling strength or the noise level is varied, which has its origin in the nonvanishing inertia.

IV. NOISE-INDUCED RESONANCE

In this section we examine the phase velocity and its power spectrum, and investigate the possibility of the noise-induced resonance. The power spectrum of the phase velocity ϕ_i is given by

$$S(f) = \frac{1}{N} \sum_{i=1}^N |\tilde{v}_i(f)|^2 \equiv \langle\langle |\tilde{v}_i(f)|^2 \rangle\rangle, \quad (7)$$

where $\tilde{v}_i(f) \equiv \int dt e^{2\pi i f t} \dot{\phi}_i$ is the Fourier component of the phase velocity at frequency f and the average over different noise realizations is also to be taken. Thus $\langle\langle \dots \rangle\rangle$ stands for

the average over the noise realizations as well as over the whole oscillators. It is related to the response function of the system via the fluctuation-dissipation theorem [19]:

$$S(f) = 2T \operatorname{Re} \chi(f), \quad (8)$$

where Re denotes the real part and the generalized susceptibility $\chi(f)$ is defined to be the Fourier transform of the appropriate linear-response function. When the system is disturbed by (time-dependent) external driving, the resulting change in the average phase velocity takes the form

$$\delta \langle \langle \tilde{v}_i(f) \rangle \rangle = \chi(f) \delta I(f), \quad (9)$$

where $\delta I(f)$ is the Fourier component of the (uniform) external driving at frequency f . In particular, the dc component of the power spectrum, describing the dc response, reads

$$S(f=0) = \frac{1}{N} \sum_{i=1}^N |\tilde{v}_i(0)|^2 = \frac{1}{N} \sum_{i=1}^N \left[\int dt \dot{\phi}_i \right]^2 \propto \langle \langle \dot{\phi}_i^2 \rangle \rangle, \quad (10)$$

where $\langle \dots \rangle$ denotes the time average. In the case of a superconducting wire network or array, the phase velocity can be identified with the voltage via the Josephson relation, and the system is driven appropriately by a (time-dependent) external current. Accordingly, Eq. (8) connects the generalized resistance with the voltage power spectrum [19].

To investigate the dc component of the power spectrum, we begin with Eq. (3) and consider two types of solutions, depending on the coupling strength: In the limit of weak coupling strength, each oscillator in the system favors oscillating with its own frequency and the system is not synchronized, yielding $\Delta \approx 0$. The solution of Eq. (3) is then given by

$$\dot{\phi}_i = \omega_i + (v_{i0} - \omega_i) e^{-t/\mu} + \frac{1}{\mu} e^{-t/\mu} \int_0^t dt' e^{t'/\mu} \eta_i(t'), \quad (11)$$

where $v_{i0} \equiv \dot{\phi}_i(t=0)$ is the initial phase velocity. Taking the time average of Eq. (11) in the stationary state ($t \rightarrow \infty$), we obtain the mean phase velocity or the frequency of the i th oscillator,

$$\langle \dot{\phi}_i \rangle = \omega_i. \quad (12)$$

We now take the average over the N oscillators; this reduces Eq. (12) to

$$\langle \langle \dot{\phi}_i \rangle \rangle \equiv \frac{1}{N} \sum_{i=1}^N \langle \dot{\phi}_i \rangle = \frac{1}{N} \sum_{i=1}^N \omega_i = 0, \quad (13)$$

where the symmetry of the distribution $g(\omega)$ about $\omega=0$ in the thermodynamic limit ($N \rightarrow \infty$) has been used.

On the other hand, the average of the square of the oscillator frequency, corresponding to the dc component of the power spectrum, does not vanish:

$$\langle \langle \dot{\phi}_i^2 \rangle \rangle = \frac{1}{N} \sum_{i=1}^N \langle \dot{\phi}_i \rangle^2 = \langle \langle \omega_i^2 \rangle \rangle, \quad (14)$$

where $\langle \langle \omega_i^2 \rangle \rangle$ corresponds to the variance of the distribution of ω_i , i.e., $\langle \langle \omega_i^2 \rangle \rangle = \int d\omega g(\omega) \omega^2$. For example, in the simple case of the δ -function distribution $g(\omega) = (1/2) \times [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$, the variance is given by $\langle \langle \omega_i^2 \rangle \rangle = \omega_0^2$, whereas for the semicircle distribution with radius r we have $\langle \langle \omega_i^2 \rangle \rangle = r^2/2$. Note that in this weak-coupling limit $\langle \langle \dot{\phi}_i^2 \rangle \rangle$ as well as $\langle \langle \dot{\phi}_i \rangle \rangle$ does not depend on the noise level T , indicating the absence of noise-induced effects.

In the limit of strong coupling strength, the oscillators tend to oscillate in a coherent manner, displaying synchronization ($\Delta \approx 1$). Since the order parameter Δ in Eq. (3) depends explicitly on the noise, decreasing with the noise level T , it is expected that unlike in the weak-coupling limit $\langle \langle \dot{\phi}_i^2 \rangle \rangle$ varies with the noise level. When the noise level is sufficiently low ($T \approx 0$) in this strong-coupling limit, the system is fully synchronized and described by the stationary solution

$$\phi_i = \sin^{-1} \left(\frac{\omega_i}{K\Delta} \right), \quad (15)$$

which yields $\langle \langle \dot{\phi}_i \rangle \rangle = 0$ and $\langle \langle \dot{\phi}_i^2 \rangle \rangle = 0$. At high noise levels ($T \rightarrow \infty$), on the other hand, the system is not synchronized ($\Delta \approx 0$), and we obtain $\langle \langle \dot{\phi}_i^2 \rangle \rangle = \langle \langle \omega_i^2 \rangle \rangle$, similarly to the case of the weak-coupling limit. Accordingly, in the strong-coupling limit, $\langle \langle \dot{\phi}_i^2 \rangle \rangle$ is expected to behave with noise as follows: At low noise levels, $\langle \langle \dot{\phi}_i^2 \rangle \rangle$ increases from zero with the noise. As the noise level is raised further, it saturates eventually toward its asymptotic value $\langle \langle \omega_i^2 \rangle \rangle$. Note that $\langle \langle \dot{\phi}_i^2 \rangle \rangle$ just corresponds to the dispersion or mean-square displacement of the oscillator frequencies over the system since $\langle \langle \dot{\phi}_i \rangle \rangle = 0$. Its monotonic growth thus indicates suppression of the frequency synchronization, which accompanies that of the phase synchronization measured by the order parameter in Sec. III. The dc susceptibility, given by $\chi_0 \equiv \operatorname{Re} \chi(f=0) = S(f=0)/2T \propto \langle \langle \dot{\phi}_i^2 \rangle \rangle / T$, then grows as the noise level T is increased from zero and diminishes with T at high noise levels; in between it is expected to reach its maximum. Therefore, in contrast with synchronization, which is suppressed by noise, the response of the phase velocity to the (uniform) external driving can be enhanced by adding an appropriate amount of noise.

To confirm the analytical argument presented above, we have performed numerical simulations on the set of equations of motion in Eq. (1). For convenience, we have considered the semicircle distribution for $g(\omega)$, and integrated Eq. (1) with discrete time steps of $\delta t = 0.01$. In computing the phase velocity, $N_t = 10^5$ time steps have been used at each run, with the data from the first 5×10^4 steps discarded. We have again varied both δt and N_t to verify that the stationary state has been achieved, and performed ten independent runs with different initial configurations, over which averages have been taken. In this manner we have computed $\langle \langle \dot{\phi}_i^2 \rangle \rangle$ in the system of N oscillators, for N up to 4096, and confirmed that there are no appreciable finite-size effects for $N \geq 1000$.

Figure 3 presents the obtained behavior of the dc susceptibility or the dc component of the noise-divided power spectrum with the noise level T in the system of $N = 2000$ oscil-

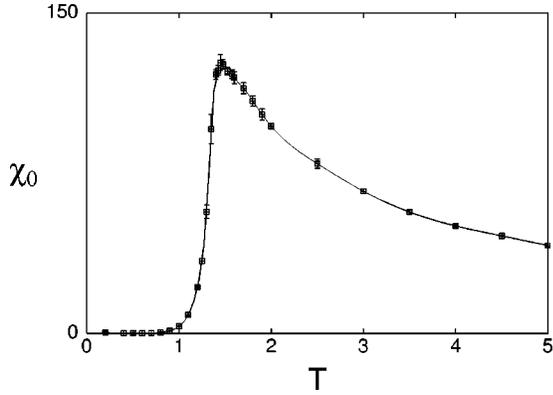


FIG. 3. Behavior of the dc susceptibility χ_0 (in arbitrary units) with the noise level T , in the case of the semicircle distribution with radius $r=0.5$, revealing the noise-enhanced response of the phase velocity. Error bars have been estimated by the standard deviation and the solid curve is merely a guide to the eye.

lators, each having inertia $\mu=0.8$. The semicircle distribution of radius $r=0.5$ has been chosen for $g(\omega)$ and the coupling strength $K=3$ adopted. In particular we have considered both cases of increasing and decreasing the noise level, only to obtain the same results within error bars. The behavior shown in Fig. 3 demonstrates that noise helps the system escape from the potential well, enhancing the response of the phase velocity to external driving. It is of interest to note that $T_m (\approx 1.4)$, at which the response becomes its maximum, is almost the same as the critical noise strength T_c below which synchronization sets in [$T_c \approx 1.4$ for $K=3$ as shown in Fig. 2(a)]. The height of the effective potential barrier of the system described by Eq. (4) is given by $K\Delta$. Since the order parameter Δ decreases with the noise level T , the barrier height also becomes lower with T , helping the escape from the potential well and enhancing the response to the external driving. Eventually, at T_c the potential barrier vanishes and the response reaches the maximum. It is thus concluded that noise not only hinders synchronization, making the critical coupling strength K_c larger [see Fig. 1(b)], but also enhances the response of the phase velocity to the external driving force.

We now investigate the ac components of the power spectrum, i.e., the power spectrum at nonzero frequencies, which gives the possibility of noise-induced intrawell resonance. For this purpose, we have also performed numerical simulations on the equations of motion, using the same parameter values, and compute the power spectrum of the phase velocity through the use of the fast Fourier transform. The obtained power spectrum as a function of the frequency f is shown in Fig. 4. At each noise level, averages have been taken over ten independent runs with different initial configurations, to obtain the data represented by such symbols as solid circles, open circles, solid squares, etc., and the error bars have been estimated by the standard deviation. Note that in the absence of noise ($T=0$), no peak appears at any finite frequencies, which is natural in the system without periodic (ac) driving. When small noise comes into the system, however, a peak develops at a nonzero frequency ($f \approx 0.3$ in our simulation results) and grows up with the noise, suggesting the activation of intrawell oscillation by noise. As the noise level is raised, the amplitude of such a noise-induced in-

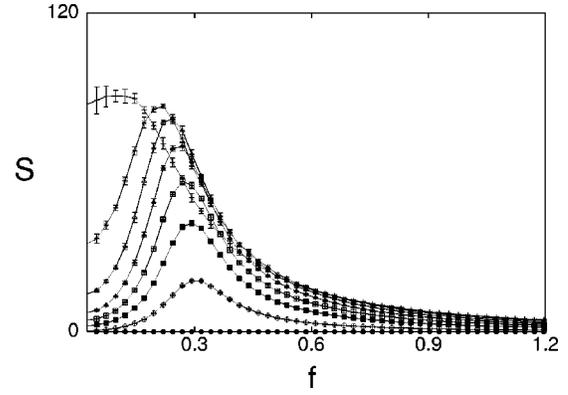


FIG. 4. Power spectrum of the phase velocity (in arbitrary units) at various noise levels: $T=0$ (solid circles), $T=0.1$ (open circles), $T=0.3$ (solid squares), $T=0.5$ (open squares), $T=0.7$ (solid triangles), $T=0.9$ (open triangles), $T=1.1$ (asterisks), and $T=1.3$ (plus signs).

trawell oscillation is expected to grow, lowering its frequency. Indeed the frequency at which the peak appears in Fig. 4 shifts toward lower values, demonstrating a noise-induced frequency shift. It eventually approaches zero frequency; this describes the system kicked by noise in a potential-well minimum and escaping from the minimum. Namely, the intrawell oscillation induced by noise turns into the interwell transition. To disclose the noise-induced effects in such intrawell motion, we have also computed the generalized susceptibility at several frequencies versus the noise level, where noise-induced enhancement in the response can again be observed. In particular, at finite frequencies, it is convenient to characterize such noise-induced effects by the appropriate quality factor

$$Q \equiv S_{max}(\delta f/f_{max})^{-1}, \quad (16)$$

where S_{max} is the peak height of the power spectrum, f_{max} is the corresponding frequency, and δf is the half-width of the peak. Thus the quality factor Q , given by the ratio of the peak height to the relative width, measures the degree of coherent motion [13]. We have computed Q from the power spectrum obtained from ten independent runs, taking the average at each noise level. The obtained behavior of the quality factor Q as a function of the noise level T is shown in Fig. 5, which demonstrates the presence of the intrawell resonance induced by noise. The value $T \approx 0.7$ at which Q reaches its maximum is apparently lower than that for the interwell motion in Fig. 3, indicating that intrawell resonance can be induced by weaker noise.

V. SUMMARY

We have studied the synchronization phenomena and the noise-induced motion in a system of globally coupled oscillators, each possessing finite inertia, subject to constant driving force. The detailed behavior of the order parameter depending on the coupling strength and the noise level has been obtained from numerical simulations, which has revealed hysteresis both with the coupling and with the noise as well as suppression of the synchronization by noise. The hysteresis with respect to the coupling is most conspicuous

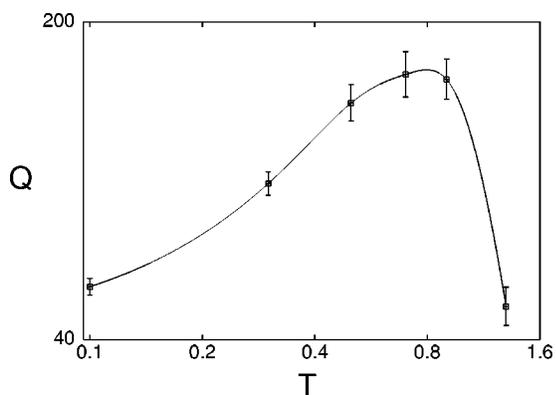


FIG. 5. Behavior of the quality factor Q with the noise level T (in the logarithmic scale), exhibiting the noise-induced resonance. Error bars have been estimated by the standard deviation and the solid curve is merely a guide to the eye.

in the absence of noise, weakening as the noise comes into the system; that with respect to the noise appears large at intermediate coupling strengths, diminishing with the coupling strength. We have also considered the power spectrum of the phase velocity, as the response of the system to the (time-dependent) external driving, and examined the possibility of the noise-induced resonance in the system. The dc component of the power spectrum, which corresponds to the dispersion of the mean oscillator frequency, has been shown to grow with noise, again manifesting suppression of the synchronization. On the other hand, the noise-divided power spectrum or the generalized susceptibility, which describes the response of the phase velocity to the external driving, has been found to display a peak at a finite noise level, revealing

the presence of noise-induced enhancement in the response. In particular, noise-induced resonance in the intrawell motion has been observed in the behavior of the quality factor with the noise strength. It is thus concluded that noise in the system of coupled oscillators not only suppresses phase synchronization but also helps the system to escape from a potential-well minimum in the response of the phase velocity, inducing the resonance. Such noise-induced resonance may be manifested by a resonance peak of the voltage power spectrum in the case of a superconducting wire network. Finally, we note that the major role of inertia is to bring about hysteresis in the response of the system. The inertia is in general necessary for the system to possess (finite) natural frequencies, and expected to be essential to the ac resonance at these finite frequencies. On the other hand, it may not be crucial in the dc resonance behavior of the power spectrum of the phase velocity in the system of coupled oscillators. Preliminary results we have obtained for the case without inertia indeed indicate that the peak at zero frequency persists whereas that at finite frequency disappears, suggesting the presence of only the interwell motion. A detailed investigation of this and other effects is left for further study.

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